Electric Vector Potential Formulation to Model a Magnetohydrodynamic Inertial Actuator

M. Mesurolle[1], Y. Lefèvre[1], C. Casteras[2]
[1]LAPLACE, University of Toulouse, CNRS, INPT-ENSEEIHT, 2 Rue C. Camichel, 31071 Toulouse Cedex 7, France
[2]CNES, Space Center of Toulouse, 18, Av. Edouard Belin, 31401 Toulouse Cedex 9, France
mael.mesurolle@laplace.univ-tlse.fr, yvan.lefevre@laplace.univ-tlse.fr

A magnetohydrodynamic (MHD) inertial actuator using a liquid metal rather than a solid mass is presented. The liquid, inside an annular channel, is accelerated thanks to transverse electric and magnetic fields. This paper proposes an electric vector potential formulation to compute the angular momentum's device under the assumption that the flow is laminar.

***Index Terms*—Magnetohydrodynamics, Electromagnetic devices, Actuators.**

I. Introduction

Most of the time, reaction and momentum wheels (DC brushless motors) are used to perform the attitude control of satellites. Despite significant improvements on electronic parts, they have many drawbacks, directly linked to their ball bearings [1]. To overcome this situation, that means using no moving solid component, a solution consists in using a conductive liquid accelerated by the Lorentz force thanks to transverse electric and magnetic fields. A MHD reaction wheel has been studied using a dynamic lumped parameter model based on a 1D steady state analytical model [2].

This paper proposes a steady state approach in order to derive an axisymmetric model of an inertial MHD actuator [3]. We use the electric vector potential approach instead of the classical electric scalar potential or the magnetic vector potential ones [4][5]. The first section presents the set of MHD equations. The second section describes the studied device. The third one concerns the steady state 2D MHD model. The last part proposes some of the obtained results by the finite difference method.

II. MHD Set of Equations

The Navier-Stokes (NS) equation describes the motion of fluids submitted to a driving force density $\mathbf{f}$. Considering an incompressible fluid, NS equation yields:

$$\rho\frac{\partial}{\partial t}\mathbf{v} + \rho(\mathbf{v}\cdot\nabla)\mathbf{v} = -\nabla p + \mu\nabla^2\mathbf{v} + \mathbf{f} \quad (1)$$

with ρ the mass density, μ the dynamic viscosity, p, the non-hydrostatic pressure and $\mathbf{v}$, the fluid speed. We use the hypothesis of quasi-static approximation, the Maxwell's equations are given by the set of equations (2):

$$\nabla.\mathbf{B} = 0 \qquad \nabla\wedge\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad \nabla\wedge\mathbf{B} = \mu_0\mathbf{j} \quad (2)$$

with $\mathbf{B}$, the magnetic field density, $\mathbf{E}$, the electric field, $\mathbf{j}$ the current density and μ_0, the magnetic permeability. Equations (1) and (2) are coupled through the Lorentz force equation and the generalized Ohm's law:

$$\mathbf{f} = \mathbf{j}\wedge\mathbf{B} \qquad \mathbf{j} = \sigma(\mathbf{E} + \mathbf{v}\wedge\mathbf{B}) \quad (3)$$

with σ the electrical conductivity. In steady state and after introducing the electric vector potential $\mathbf{T}$, following equations are obtained from equations (1)-(3):

$$\nabla\wedge(\nabla\wedge\mathbf{T}) = \sigma.\nabla\wedge(\mathbf{v}\wedge\mathbf{B}) \quad (4)$$

$$\rho(\mathbf{v}\cdot\nabla)\mathbf{v} = -\nabla p + \mu\nabla^2\mathbf{v} + (\nabla\wedge\mathbf{T})\wedge\mathbf{B} \quad (5)$$

(4)-(5) constitute the MHD set of equations at steady state.

III. Studied Device

The studied device is composed by an annular channel, with rectangular section. A liquid metal can flow in it. A DC current I is imposed at each electrode creating an axial electric field, as illustrated on Fig. 1. A DC radial magnetic field $\mathbf{B_0}$ is imposed by means of an electromagnet for instance.

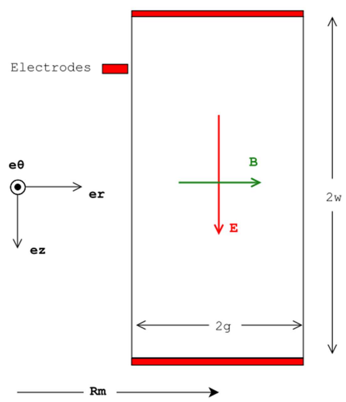

Fig. 1. A cross section of the annular channel

IV. Axisymmetric Modelling

The geometry and the magnetic field are invariant by rotation around the axis Oz. 2D axisymmetric assumptions can be applied. In this case, there is no pressure gradient along $\boldsymbol{e_\theta}$ and all variables do not depend on θ. Therefore the speed is assumed to be ortho-radial and is of the form:

$$\mathbf{v} = v(r,z)\mathbf{e_\theta} \quad (6)$$

Due to Ampere's law, the current density induces a magnetic field density **b.** As (O, r, z) is a plane of symmetry for the current density, **b** is ortho-radial. Assuming that the imposed magnetic flux density is radial and divergence free the magnetic field density **B** in the channel is given by:

$$\mathbf{B}(\mathrm{r},\mathrm{z}) = \frac{\mathrm{B_m R_m}}{\mathrm{r}}.\mathbf{e_r} + \mathrm{b}(\mathrm{r},\mathrm{z}).\mathbf{e_\theta} \quad (7)$$

B_m is the imposed magnetic flux density at the middle of channel $(r = R_m)$. In the final paper, it will be shown that the electric vector potential can be taken equal to $\frac{\mathbf{b}}{\mu_0}$

These assumptions and equations (4)-(5) yield:

$$\Delta \mathrm{b} - \frac{\mathrm{b}}{\mathrm{r}^2} + \mu_0 \sigma \mathrm{B_m R_m} \frac{\partial}{\partial \mathrm{r}}\left(\frac{\mathrm{v}}{\mathrm{r}}\right) = 0 \quad (8)$$

$$\Delta \mathrm{v} - \frac{\mathrm{v}}{\mathrm{r}^2} + \frac{\mathrm{B_m}}{\mu.\mu_0}\frac{\mathrm{R_m}}{r^2}\frac{\partial}{\partial r}(\mathrm{rb}) = 0 \quad (9)$$

To these equations, the boundary conditions must be added. The fluid has no velocity at the vertical wall and at the horizontal electrodes:

$$\mathrm{v}(\mathrm{Rm} \pm \mathrm{g}, \mathrm{z}) = 0 \quad (10)$$

$$\mathrm{v}(\mathrm{r}, \pm \mathrm{w}) = 0 \quad (11)$$

At the vertical non-conductive walls the current density is tangential so that the axial gradient of b is equal to zero:

$$\frac{\partial}{\partial z} b(r, \pm w) = 0 \quad (12)$$

By calculating the circulation of the current density through an horizontal section at z vertical position and using Stoke's theorem we have:

$$\mathrm{b}(R_m - \mathrm{g}, \mathrm{z}) = 0 \quad (13)$$

$$\mathrm{b}(R_m + g, \mathrm{z}) = \frac{\mu_0}{2\pi(\mathrm{R_m} + \mathrm{g})} I_0 \quad (14)$$

where $\mathrm{I_0}$ is the imposed electrical current through the horizontal electrodes. $\mathrm{I_0}$ and $\mathrm{B_m}$ control the flow velocity and others electromechanical characteristics as the torque. In the classical formulation with the electric scalar potential, it is an electrical potential drop U_0 between the two electrodes instead of the electrical current I_0 that would have been imposed.

V. SOME RESULTS

The axisymmetric steady state model is constituted by the coupled equations (8) and (9) and the boundary conditions (10)-(14). As the study domain is very simple (Fig. 1), finite difference method is chosen to discretize the study domain and these partial derivatives equations. The study domain of Fig. 1 is then discretized by a grid of i_m vertical lines and of j_m horizontal lines.

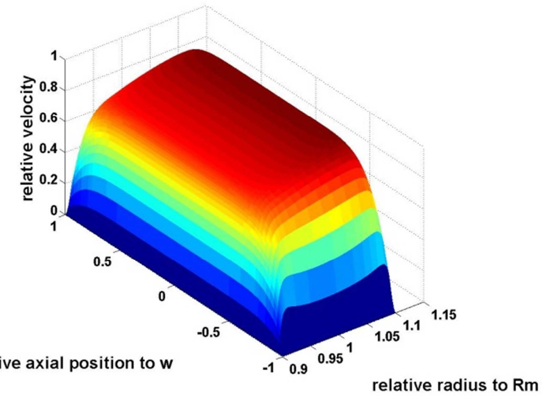

Fig. 2: Distribution of velocity in the study domain

The type of results is shown on Fig. 2. From the distribution of velocity and the induced magnetic field density in the study domain the electromechanical quantities can be computed. Some of them are compared to the ones obtained from the 1D model presented in [3] on Table I.

TABLE I
COMPARISON OF 1D AND 2D MODELS

	1D model	2D model
Torque	0.0043 N.m	0.0044 N.m
Mean velocity	0.229 m/s	0.227 m/s
Angular Momentum	0.1962 kg.m^2/s	0.1984 kg.m^2/s

VI. CONCLUSION

We propose a new formulation to model a MHD inertial actuator by using the electric vector potential instead of the classical electric scalar potential. This formulation is well adapted when the current is imposed instead of the electric potential drop. In the final paper, equations (8) and (9) will be explicitly deduced from (4) and (5). The evaluated electromechanical quantities like torque, angular momentum Joule losses and voltage drop will be shown.

VII. ACKNOWLEDGEMENT

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